# Numerical model for femtosecond pulse propagation in hollow core fibers 

Valer Tosa ${ }^{1}$, Anna Gabriella Ciriolo ${ }^{2}$, Rebeca Martinez Vazquez $^{2}$, Caterina Vozzi², and Salvatore Stagira ${ }^{3}$

${ }^{1}$ National Institute for R\&D Isotopic and Molecular Technologies, INCDTIM Cluj Napoca, Romania ${ }^{2}$ Institute for Photonics and Nanotechnologies (IFN), National Research Council (CNR), Milano, Italy ${ }^{3}$ Politecnico di Milano, Physics Department, Milano, Italy

Abstract. A numerical model used to obtain pulsed field configuration along and across a hollow core dielectric waveguide filled with an ionizing gas and operated as a device for high harmonics generation is presented. The model was developed for an arbitrary gas density profile and arbitrary fiber diameter variation. The results of the calculation were tested against experimental measurements and excellent agreement was obtained for the fluorescence emission along the waveguide.

## The free space propagation

$\nabla^{2} E_{1}(r, z, t)-\frac{1}{c^{2}} \frac{\partial^{2} E_{1}(r, z, t)}{\partial t^{2}}=\frac{\omega^{2}}{c^{2}}\left(1-\eta_{e f f}^{2}\right) E_{1}(r, z, t)$
$\eta_{e f f}\left(n_{a}, n_{e}, r, z, t\right)=\eta_{0}\left(n_{a}\right)+\eta_{2}\left(n_{0}\right) I(r, z, t)-\frac{\omega_{p}^{2}\left(n_{e}, r, z, t\right)}{2 \omega^{2}}$
After going to moving frame $z^{\prime}$, paraxial approx. and FT :
$\nabla_{\perp}^{2} \widetilde{E_{1}}\left(r, z^{\prime}, \omega\right)-\frac{2 i \omega}{c} \frac{\partial \widetilde{E_{1}}\left(r, z^{\prime}, \omega\right)}{\partial z^{\prime}}=\widetilde{G}\left(r, z^{\prime}, \omega\right)$




Measured emission of Ar species along propagation C.A. Froud et al., J. Opt. A 11054011 (2009)


Nonlinear Schrodinger equation:
R.T Chapman et al, Opt. Express 1813279 (2010)

Sinear step

$$
\begin{aligned}
& \nabla_{\perp}^{2} \widetilde{E_{1}}\left(r, z^{\prime}, \omega\right)-\frac{2 i \omega}{c} \frac{\partial \widetilde{E_{1}}\left(r, z^{\prime}, \omega\right)}{\partial z^{\prime}}=0 \\
& \widetilde{E_{1}}\left(r, z^{\prime}, \omega\right)=\sum_{j} b_{j}\left(z^{\prime}, \omega\right) J_{0}\left(\mu_{j} r / a\right) \exp \left(i \int_{0}^{z^{\prime}} \gamma_{j}(z) d z\right)
\end{aligned}
$$

$$
b_{j}=\int_{0}^{a} r J_{0}\left(\mu_{j} \frac{r}{a}\right) E_{1}(r) d r
$$

$$
b_{j}\left(z^{\prime}+\Delta z^{\prime}\right)=b_{j}\left(z^{\prime}\right) \cdot \exp \left(\kappa_{j} \Delta z^{\prime}-\alpha_{j} \Delta z^{\prime}\right)
$$

$$
\widetilde{E_{1}}\left(r, z^{\prime}+\Delta z^{\prime}, \omega\right)=\sum_{j} b_{j}\left(z^{\prime}+\Delta z^{\prime}, \omega\right) J_{0}\left(\mu_{j} r / a\right)
$$

The guided propagation Nou-linear step


On-axis field in frequency and time


