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Quantum transport through a quantum dot coupled to a Majorana ring

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INTRODUCTION

- We theoretically investigate quantum transport through a quantum dot coupled to Majorana bound states connected at the ends of a topological superconductor nanowire threaded by a tunable magnetic flux. Majorana bound states are fermions that are their own antiparticles. Quantum dots are semiconductor nanocrystals with diameters ranging from one nanometer to a few tens of nanometers. In these nanostructures the motion of particles is confined in all three spatial directions. These particles have discrete energy levels similarly to the single atom. The nonequilibrium Green's function formalism is used to derive the current formulas with the spectral function of the quantum dot, linear and differential conductances through the quantum system. The retarded Green's functions are determined by applying the equation of motion method.
- We show that when the two Majorana bound states do not overlap, the linear conductance has a 2π periodicity as a function of magnetic flux phase, independent of the quantum dot energy, or the finite values of dot-Majorana couplings. The spectral function of the quantum dot exhibiates an usual Majorana bound state-induced spectrum charachteristic.

I. THEORETICAL MODEL

Anderson Hamiltonian:

$$\mathcal{H} = \mathcal{H}_{\text{Leads}} + \mathcal{H}_{\text{MBS}} + \mathcal{H}_{\text{D}} + \mathcal{H}_{\text{T}}$$

$\mathcal{H}_{\text{Leads}}$ – Metallic electrodes Hamiltonian
$\mathcal{H}_{\text{MBS}}$ – Majorana Bound States Hamiltonian
$\mathcal{H}_{\text{D}}$ – Quantum Dot Hamiltonian
$\mathcal{H}_{\text{T}}$ – Tunneling Hamiltonian
$\mu_{L(R)}$ – chemical potential in lead $L(R)$
$\Gamma_{L(R)}$ – coupling strength between QD and leads

The temperatures in leads: $T_L = T_R = T$

Fermi-Dirac function for electrons (holes):

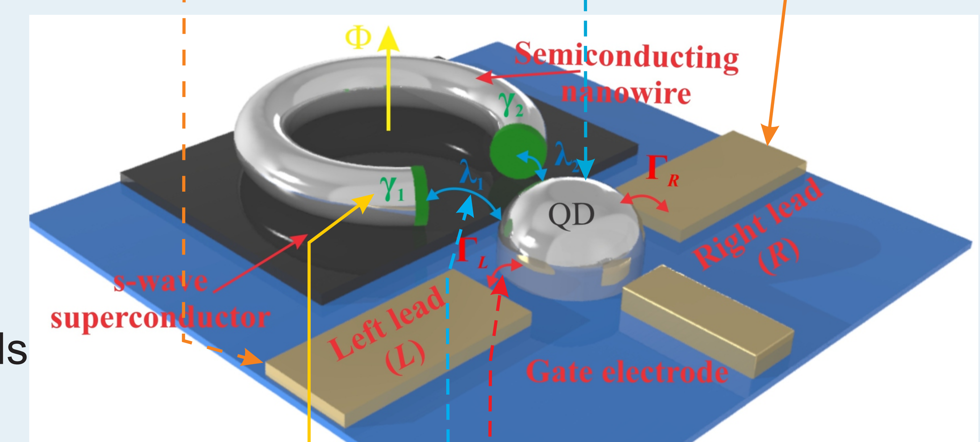
$$f^e_\alpha(\varepsilon) = 1/[e^{(\varepsilon-\mu_\alpha)/T}+1]$$
$$f^h_\alpha(\varepsilon) = 1 - f^e_\alpha(-\varepsilon) \quad \hbar = k_B = 1$$

Overlap energy: $\varepsilon_M \propto e^{-L/\xi}$; L - loop length ; ξ - superconducting coherence length

Regular fermion operators: $\gamma_1 = (f^\dagger + f)/\sqrt{2}$, $\gamma_2 = i(f^\dagger - f)/\sqrt{2}$

$$\mathcal{H}_{\text{Leads}} = \sum_{\alpha,k} \varepsilon_{\alpha k} c^\dagger_{\alpha k} c_{\alpha k} \qquad \varepsilon_{\alpha k} = \varepsilon_k - \mu_\alpha, \ \alpha \equiv \text{left } (L), \text{right } (R)$$

$$\mathcal{H}_{\text{D}} = \varepsilon_d d^\dagger d$$



$$\mathcal{H}_{\text{MBS}} = i\varepsilon_M \gamma_1 \gamma_2 \qquad \mathcal{H}_{\text{MBS}} = \varepsilon_M \left(f^\dagger f - \frac{1}{2}\right)$$

$$\mathcal{H}_{\text{T}} = [(\lambda_1 d - \lambda_1^* d^\dagger)\gamma_1 + i(\lambda_2 d + \lambda_2^* d^\dagger)\gamma_2] + \sum_{\alpha,k}(V_{\alpha k} c^\dagger_{\alpha k} d + V^*_{\alpha k} d^\dagger c_{\alpha k})$$

$\lambda_1 = |\lambda_1| e^{i\phi/4}$, $\lambda_2 = |\lambda_2| e^{-i\phi/4}$

Magnetic flux phase: $\phi = \pi\Phi/\Phi_0 = 2\arg(\lambda_1/\lambda_2)$, $\Phi_0 = h/(2e)$

II. CURRENT FORMULAS

Current in lead α: (Normal tunneling current; Andreev reflection current)

$$I_\alpha = \frac{e}{h}\int d\varepsilon \{\mathcal{T}^{ee}_{\alpha\alpha'}(\varepsilon)[f^e_\alpha(\varepsilon) - f^h_\alpha(\varepsilon)] + \mathcal{T}^{eh}_{\alpha\alpha}(\varepsilon)[f^e_\alpha(\varepsilon) - f^h_{\alpha'}(\varepsilon)]\}$$

In steady state: $I_L + I_R = 0 \rightarrow I = I_L = -I_R \rightarrow I = \frac{1}{2}(I_L - I_R)$

Transmission coefficient through the leads:

$$\mathcal{T}^{ee}_{\alpha\alpha'}(\varepsilon) = \Gamma^e_\alpha \Gamma^e_{\alpha'} |G^r_{d11}(\varepsilon)|^2$$

Local Andreev reflection:

$$\mathcal{T}^{eh}_{\alpha\alpha}(\varepsilon) = \Gamma^e_\alpha \Gamma^h_\alpha |G^r_{d12}(\varepsilon)|^2$$

$G^r_d(\varepsilon)/G^a_d(\varepsilon)$ - retarded / advanced Green's function for the QD; $G^a_d(\varepsilon) = [G^r_d(\varepsilon)]^\dagger$

Nambu representation

$$G^r_d(\varepsilon) = \begin{pmatrix} G^r_{d11}(\varepsilon) & G^r_{d12}(\varepsilon) \\ G^r_{d21}(\varepsilon) & G^r_{d22}(\varepsilon) \end{pmatrix}$$

Electron / hole coupling

$$\mathbf{\Gamma}_\alpha = \begin{pmatrix} \Gamma^e_\alpha & 0 \\ 0 & \Gamma^h_\alpha \end{pmatrix} \qquad \Gamma^{e(h)}_\alpha = 2\pi \sum_k |V_{\alpha k}|^2 \delta(\varepsilon \mp \varepsilon_{\alpha k})$$

In the wide-band limit: $\Gamma_\alpha = \Gamma$, $\Gamma^e_\alpha = \Gamma^h_\alpha = \Gamma_\alpha$

Total current through the dot

$$I = \frac{e}{h}\int d\varepsilon \mathcal{A}_d(\varepsilon)[f^e_L(\varepsilon) - f^e_R(\varepsilon)]$$

Spectral function for the dot

$$\mathcal{A}_d(\varepsilon) = i\frac{\Gamma}{2}[\mathbf{G}^>_d(\varepsilon) - \mathbf{G}^<_d(\varepsilon)]_{11}$$

III. NUMERICAL RESULTS

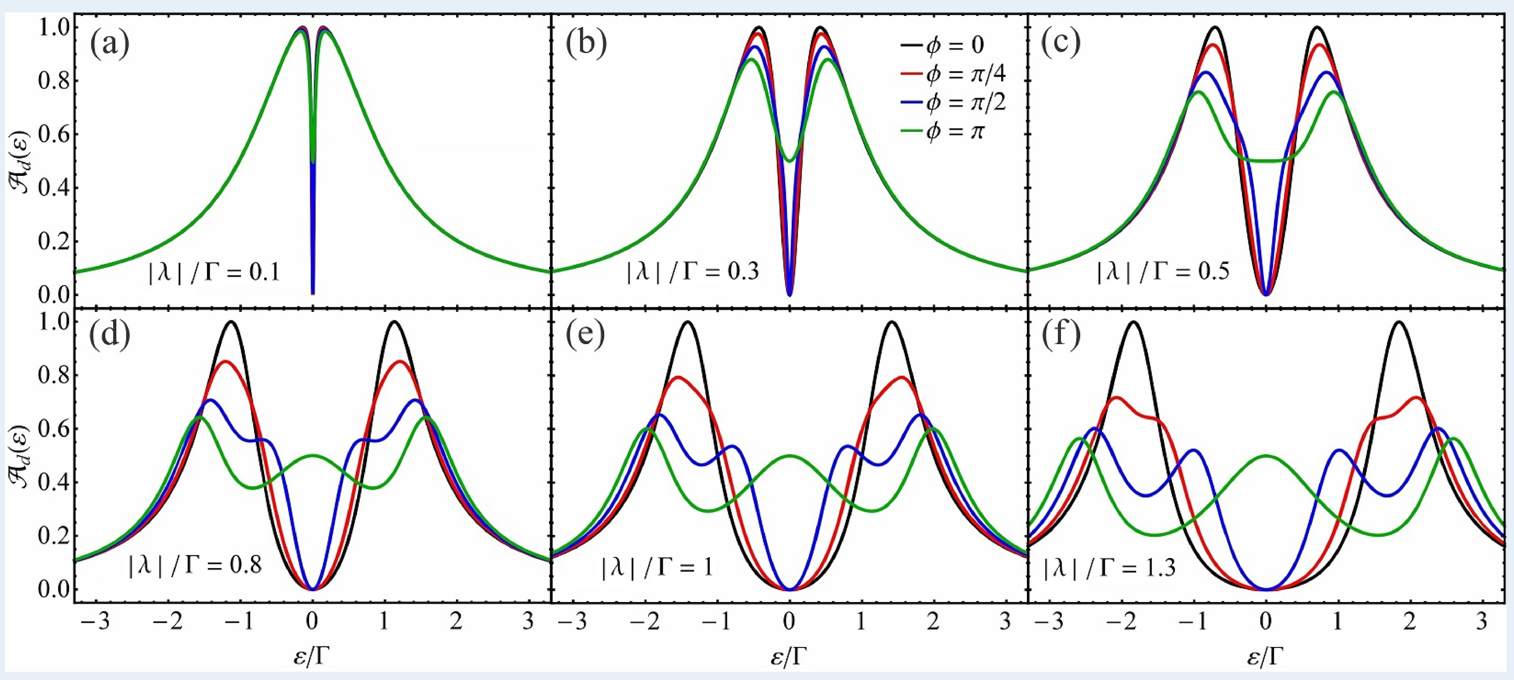


Fig. 1. Equilibrium spectral function ($eV = 0\,\Gamma$) at $\varepsilon_d = \varepsilon_M = 0\,\Gamma$ for different values of Φ and $|\lambda|$.

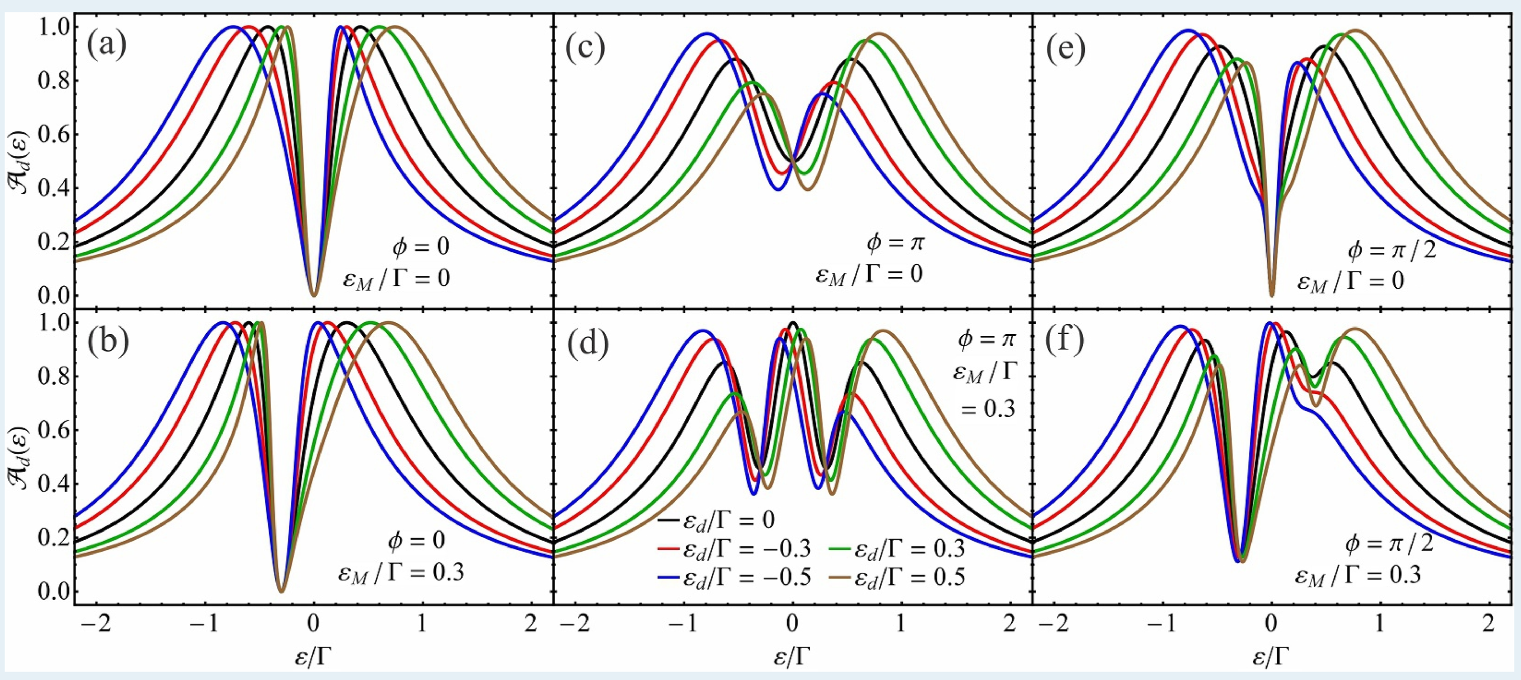


Fig. 2. Equilibrium spectral function ($eV = 0\,\Gamma$) for different values of Φ, ε_d and ε_M with $|\lambda| = 0.3\,\Gamma$.

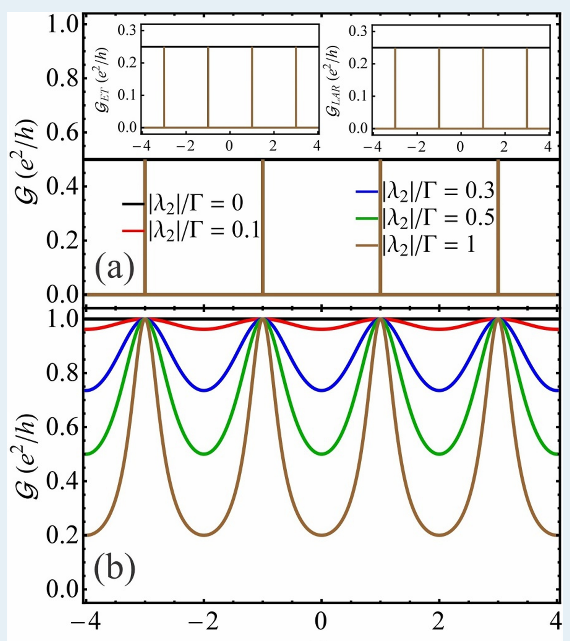


Fig. 3. Linear conduc. at $T = \varepsilon_d = 0\,\Gamma$: (a) $\varepsilon_M = 0\,\Gamma$ and (a) $\varepsilon_M = 0.3\,\Gamma$.

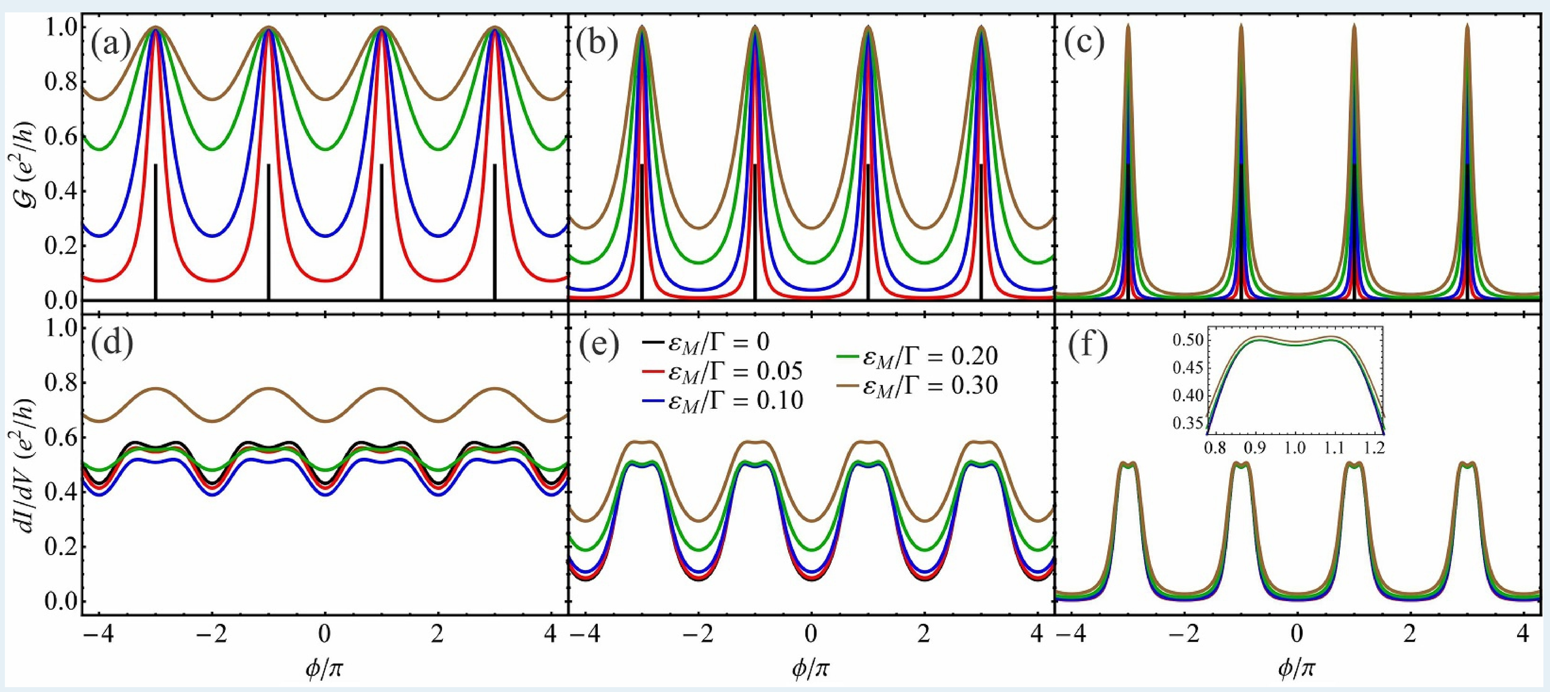


Fig.4. (a)-(c) Linear conductance vs Φ. (d)-(f) and differential conductance ($eV = 0.28\,\Gamma$) vs Φ at $\varepsilon_d = 0\,\Gamma$ for different values of ε_M with $|\lambda| = 0.3\,\Gamma$.

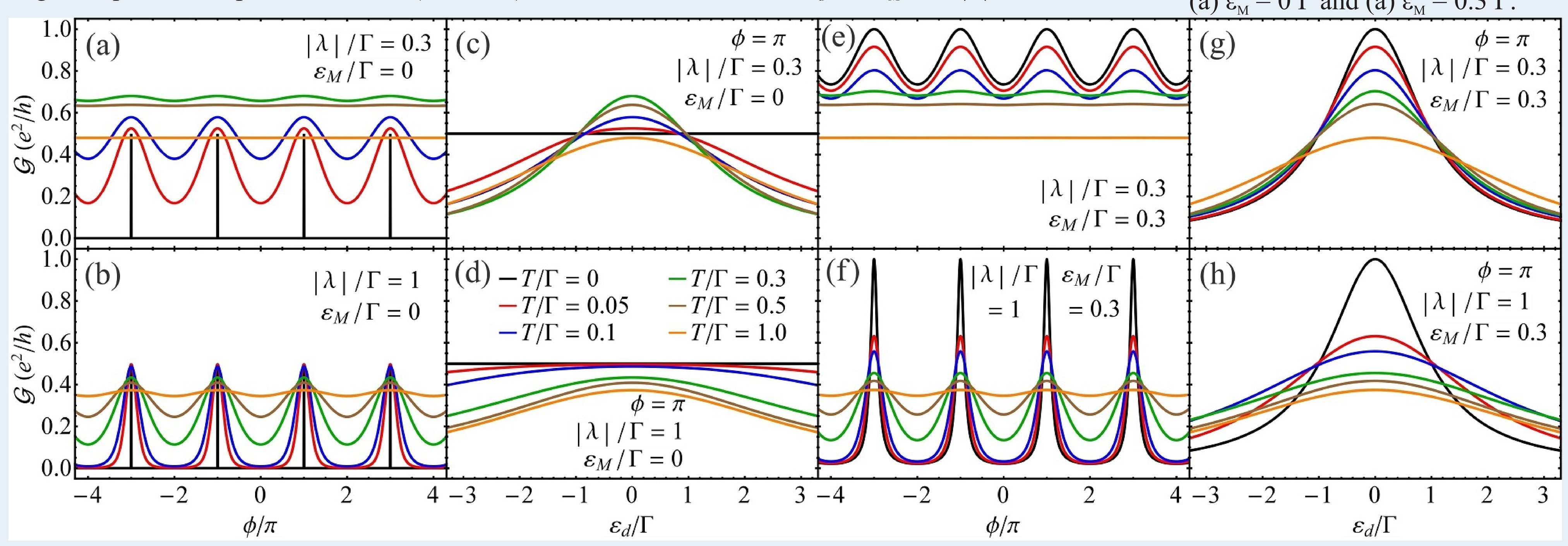


Fig. 5. Linear conductance vs Φ at $\varepsilon_d = 0\,\Gamma$ for different values of T, ε_M and $|\lambda|$.

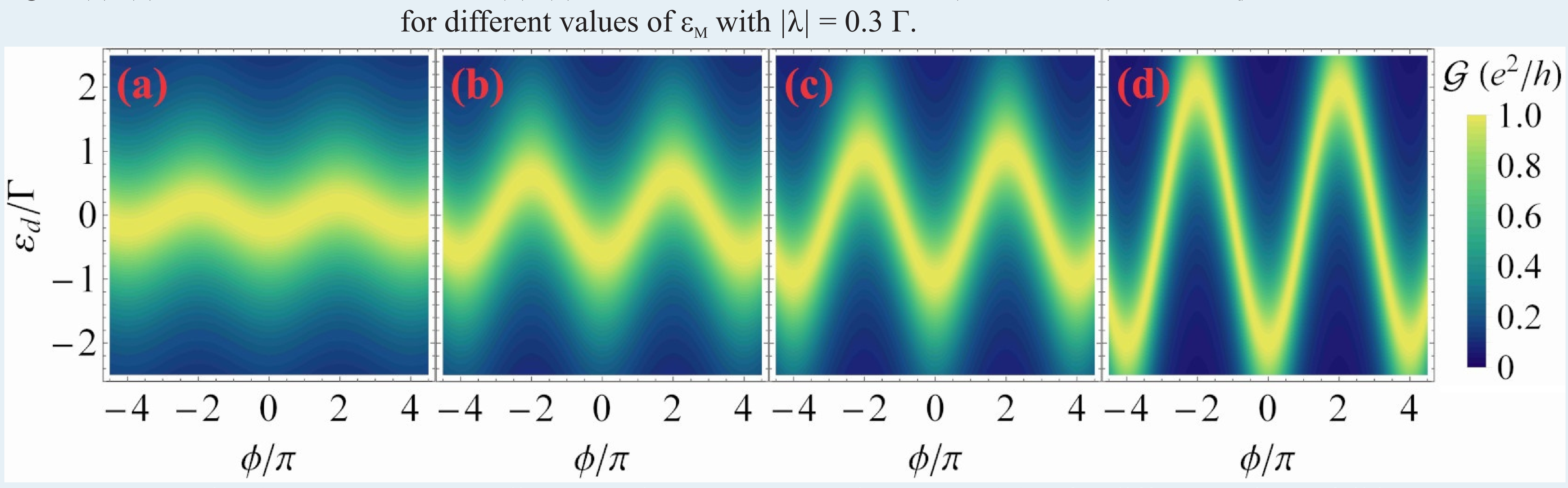


Fig. 6. Zero-temperature linear conductance vs Φ and ε_d at $\varepsilon_M = 0.3\,\Gamma$ for different values of $|\lambda|$. (a) $|\lambda| = 0.1\,\Gamma$, (b) $|\lambda| = 0.3\,\Gamma$, (c) $|\lambda| = 0.5\,\Gamma$ and (d) $|\lambda| = 1\,\Gamma$.

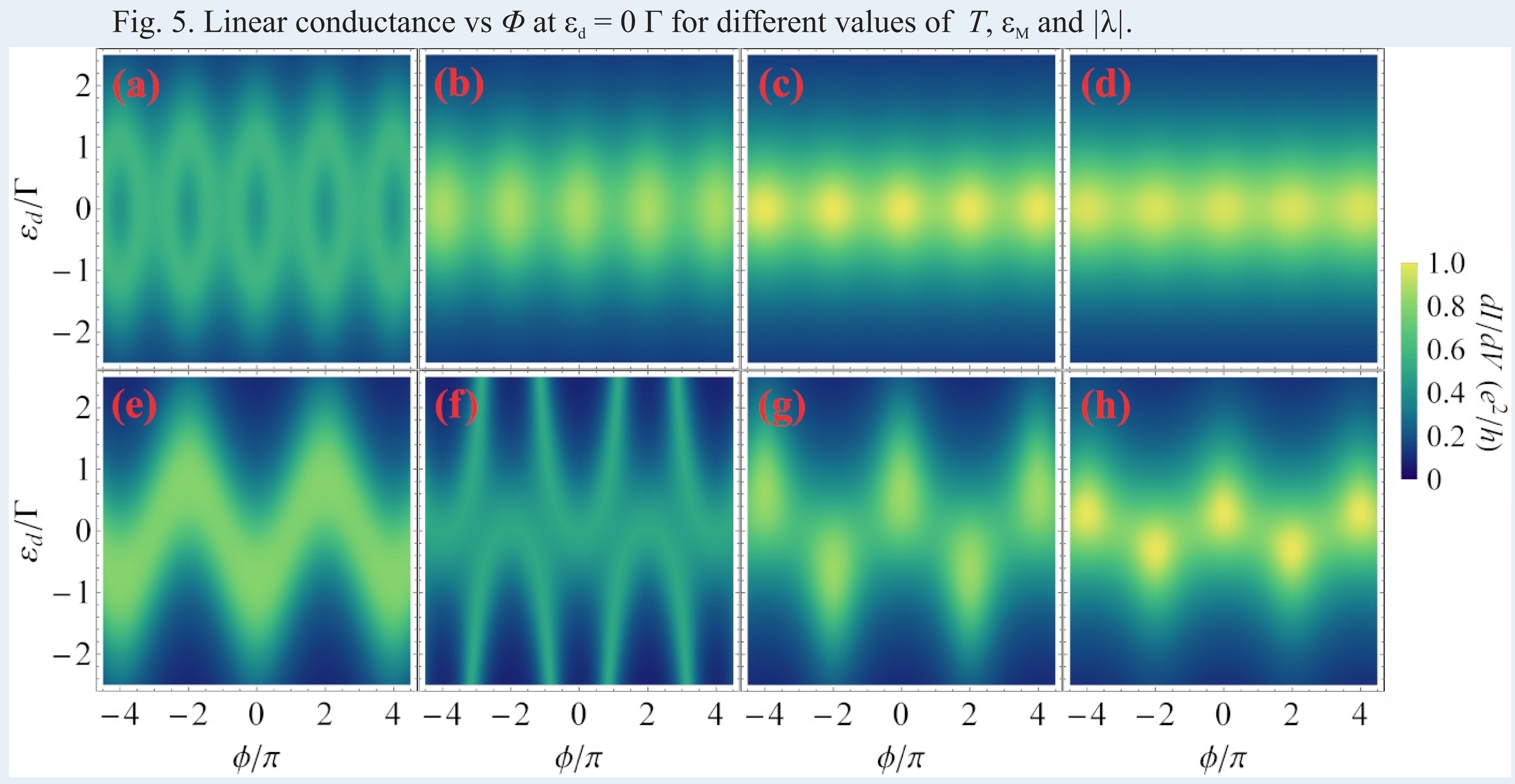


Fig. 7. Diff. cond. vs Φ and ε_d at $T = 0\,\Gamma$ and $|\lambda| = 0.3\,\Gamma$ for different values of eV. (a)-(d) $\varepsilon_M = 0\,\Gamma$ and (e)-(h) $\varepsilon_M = 0.3\,\Gamma$. (a), (e) $eV = 0.28\,\Gamma$, (b), (f) $eV = 0.55\,\Gamma$, (c), (g) $eV = 0.85\,\Gamma$ and (d), (h) eV $= 1.05\,\Gamma$.

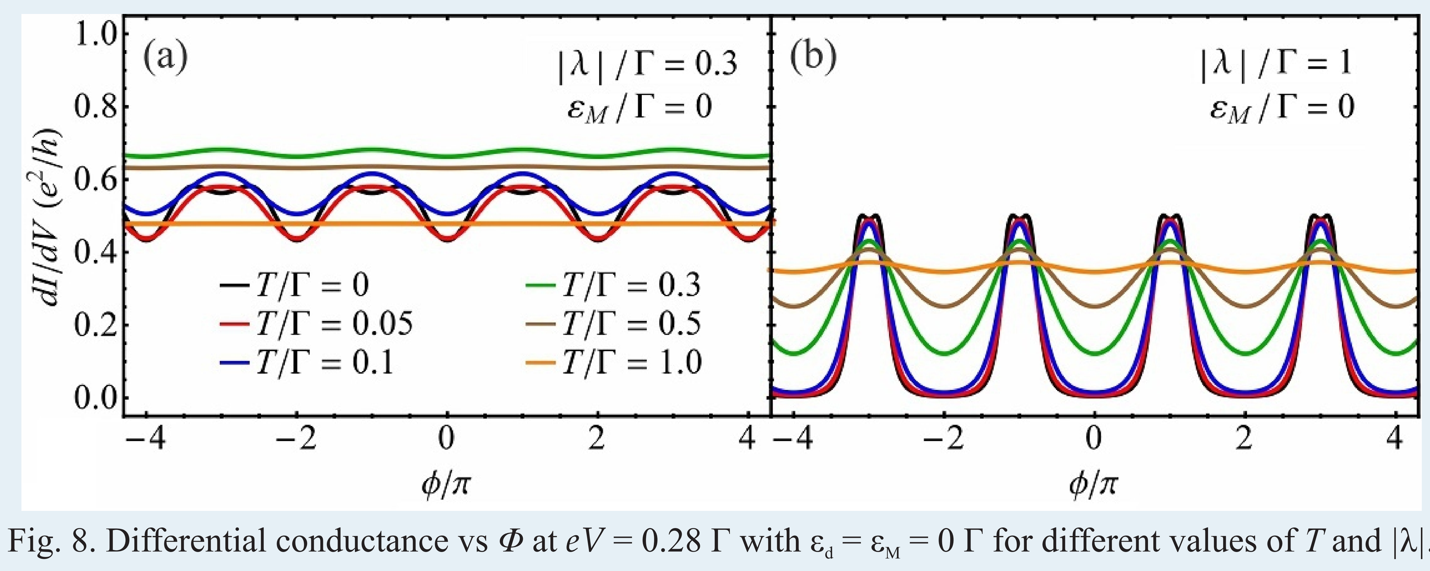


Fig. 8. Differential conductance vs Φ at $eV = 0.28\,\Gamma$ with $\varepsilon_d = \varepsilon_M = 0\,\Gamma$ for different values of T and $|\lambda|$.

V. DISCUSSION

- when the two Majorana bound states do not overlap, the **linear conductance** has a **2π** periodicity as a function of magnetic flux phase, independent of the quantum dot energy, or the finite values of dot-Majorana couplings.
- when the Majorana bound states overlap, the **linear conductance** periodicity transforms to **4π** due to **dot level energy** which is tuned away from the Fermi level. The **differential conductance** periodicity changes from **2π** to **4π** when the Majorana bound states are not perfectly degenerate and the dot level is tuned.

REFERENCES

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ACKNOWLEDGMENTS

Financial support from a grant of the Ministry of Research, Innovation and Digitization, CNCS/CCCDI-UEFISCDI, under project numbers PN-III-P1-1.1-TE-2019-0423 within PNCDI III.